A Work Project presented as part of the requirements for the Award of a Master's degree in Finance from the Nova School of Business and Economics.

Banco Invest Field Lab on Option Volatility Models

Study on the Quality of the Heston-Nandi closed-formula

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Introduction

The Banco Invest project started long before the master thesis semester even began. A first meeting was held by the end of July to better understand the specifications of the project and the final goal. The group spent the following month gathering all the information needed to develop the various models in the study. Second step, was to divide the group was divided in two, covering the two ways of solving the problem, either with GARCH models or with the Heston model. I went to work on the GARCH models, as most of my research was about it. Moreover, since I'm proficient in using Excel and with expertise in programming, I took a very practical approach to the project. Creating and designing most of the excels used in the models to fit the specifications required, as well as adapting existing structures, provided by some papers that were used in the project, just like in "*Option pricing models and volatility using Excel-VBA" and the original Heston-Nandi paper, Q*A closed-form GARCH option valuation model.". Additionally, considering the vast data collection and models being performed for the work, designing a way to automate most of the processes was one of the challenges. Moreover, some processes and calculations were too computationally intensive to use in Excel, consequently, the use of MATLAB was necessary to correctly run the models over time and to reduce the dependence on Excel, especially on Monte Carlo and for a part of the Heston-Nandi. Furthermore, MATLAB was also the language being used by Banco Invest, making it easier to implement part of the project afterward.

All things considered; I took a very practical approach to the project due to my programming knowledge. Therefore, my paper is about how to optimize the Heston-Nandi model to suit the Bank, considering the computational constraints and the competitiveness of their models.

As one of the main constraints of Banco Invest is their computational power and time to process all the required activities to maintain their activity, there is a need to keep the models as simple as possible and at the same time competitive. Since one of the suggestions was to perform the Heston-Nandi model in avoidance of the implied volatility, the first part of the paper studies the effectiveness and deviation of the closed-form formula purposed by Heston-Nandi, against the usage of Monte Carlo. As the latter is more computationally intensive but regarded by some, as more efficient.

Moreover, a significance test will be performed to the Monte Carlo against the Closed-form, in order to understand the quality of the results gives by the closed-form, as more simulations are added to the Monte Carlo, becoming more accurate and precise in terms of estimations.

Straight comparison between the closed-form and Monte Carlo simulations

The Heston-Nandi Model estimates 5 different parameters using historical data, which are used to calculate an option pricing for a desired moneyness and maturity of the option. Instead of using Monte Carlo simulation, which was the typical way to solve the model created by Duan in 1995, Heston and Nandi went a step further and derived a closed-form option pricing method, which is showed by the equation below.

$$C = \frac{1}{2}S_t + \frac{e^{-r(T-t)}}{\pi}\int_0^\infty Re\left[\frac{K^{-i\phi}f(i\phi+1)}{i\phi}\right]d\phi - Ke^{-r(T-t)}\left(\frac{1}{2}+\frac{1}{\pi}\int_0^\infty Re\left[\frac{K^{-i\phi}f(i\phi)}{i\phi}\right]d\phi\right)$$

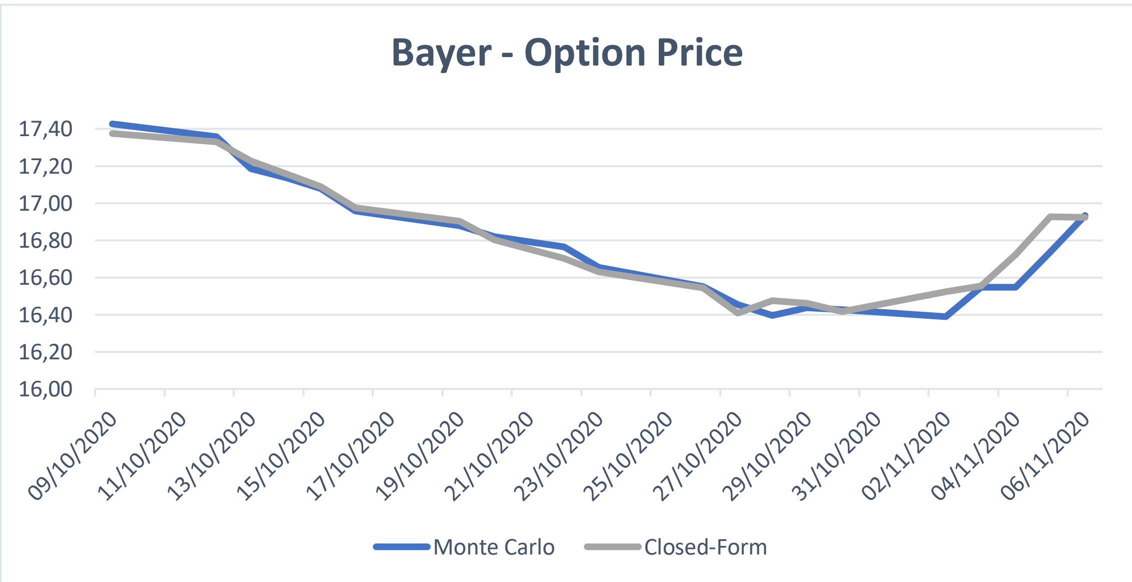
Despite this, many other authors presented the idea that, being more time and computationally intensive, if possible, the Monte-Carlo method could be used to achieve a more precise pricing option. When the real market prices are not available, it is beneficial to create the path of the volatility until the day of maturity (Kamiński 2013)(5).

Authors such as Christoffersen, P., Jacobs, K. and Ornthanalai, C declare that European call options can be calculated by "*simulating the risk-neutral return process*

R(t) and computing the sample analogue of the discounted risk neutral expectation" (Christoffersen et al. 2013)(2). According to this paper, numerical techniques provide a slightly more efficient result. Therefore, Christofferson's paper suggests a Monte-Carlo simulation method to maximize the efficiency of the Heston-Nandi model (Christoffersen et al. 2013).

However, Banco Invest faces a problem of computational power and desires a way to compute options volatility, in the most efficient method possible with the lowest computational requirements. With this in mind, to understand if the differences between Monte Carlo and the closed-form purposed by Heston-Nandi are relevant, a study using Bayer and Qualcomm will be conducted.

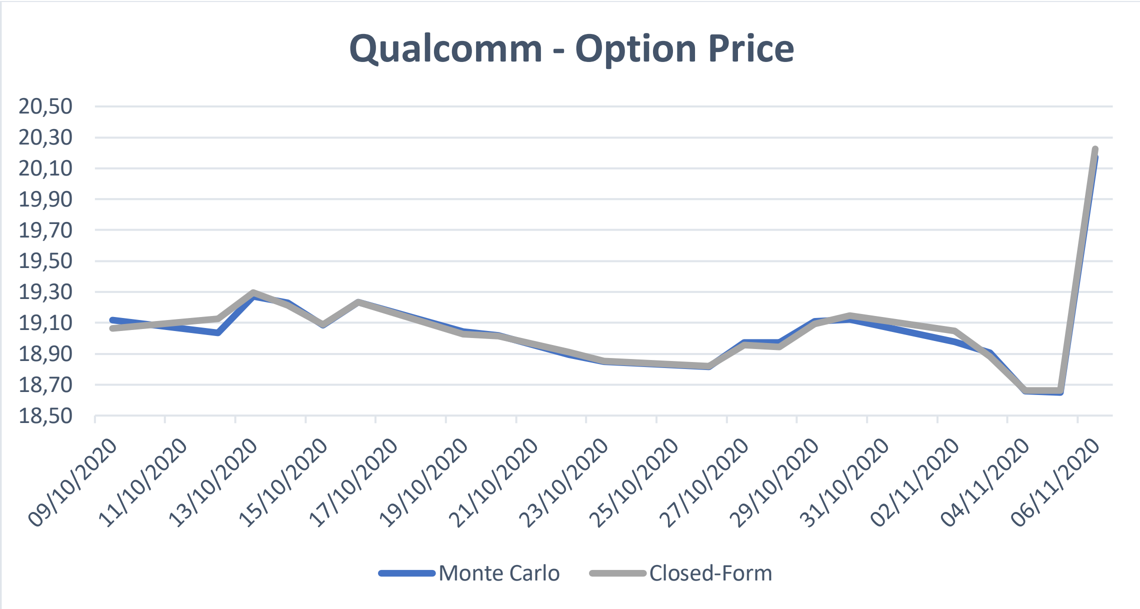
The graph and table below show the absolute difference between using Monte Carlo against the Heston-Nandi closed-form for the Bayer stock, and the same difference in terms of volatility, reverse-engineered using the Black & Scholes formula.



Dates	Option Pricing			Volatility		
	Monte Carlo	Closed-Form	Difference	Monte Carlo	Closed-Form	Difference
09/10/2020	17,43€	17,37€	0,05	44,03%	43,90%	0,13
12/10/2020	17,36€	17,33€	0,03	43,86%	43,79%	0,07
13/10/2020	17,19€	17,23€	0,04	43,42%	43,52%	0,1
14/10/2020	17,14€	17,16€	0,02	43,29%	43,35%	0,06
15/10/2020	17,08€	17,09€	0,01	43,14%	43,16%	0,02
16/10/2020	16,96€	16,97€	0,02	42,83%	42,87%	0,04
19/10/2020	16,88€	16,90€	0,02	42,63%	42,69%	0,06
20/10/2020	16,82€	16,80€	0,02	42,48%	42,44%	0,04
22/10/2020	16,76€	16,70€	0,06	42,33%	42,18%	0,15
23/10/2020	16,65€	16,63€	0,03	42,06%	41,99%	0,07
26/10/2020	16,55€	16,54€	0,01	41,79%	41,77%	0,02
27/10/2020	16,45€	16,41€	0,05	41,54%	41,42%	0,12
28/10/2020	16,40€	16,48€	0,08	41,39%	41,60%	0,21
29/10/2020	16,44€	16,46€	0,02	41,50%	41,56%	0,06
30/10/2020	16,43€	16,42€	0,01	41,47%	41,45%	0,02
02/11/2020	16,39€	16,52€	0,13	41,37%	41,72%	0,35
03/11/2020	16,55€	16,55€	0,01	41,78%	41,80%	0,02
04/11/2020	16,55€	16,72€	0,18	41,78%	42,23%	0,45
05/11/2020	16,74€	16,93€	0,19	42,26%	42,76%	0,5
06/11/2020	16,93€	16,92€	0,01	42,77%	42,74%	0,03
Average	16,78€	16,80€	0,0231	42,38%	42,44%	0,06
Stand. Dev	0,326	0,311		0,837	0,797	

Both strategies, Monte Carlo and Closed-Form seem to follow the same path and the difference between them are on average about 0,02€ in terms of option pricing and about 0,06% in terms of volatility. These values are very similar to each other, with few decimal deviations at the worst, something that adjusts eventually. Concluding in average volatility for the period of 42,38% for the Monte Carlo, against the 42,44% for the Closed-Formula. Additionally, it is important to clarify, that since Monte Carlo gives a randomness approach to the option pricing, slight deviations appear by running the model again, which seems nonsignificant. For this test, the Monte Carlo simulation iterated 10,000 times for each date of the observation.

In order to understand if the path taken from Bayer is not biased to the stock, the same study is conducted on Qualcomm as previously mentioned.



Dates	*Option Pricing*			*Volatility*		
	Monte Carlo	Closed-Form	Difference	Monte Carlo	Closed-Form	Difference
09/10/2020	19,12$	19,06$	0,05	48,39%	48,25%	0,14
12/10/2020	19,03$	19,12$	0,09	48,17%	48,41%	0,24
13/10/2020	19,27$	19,30$	0,03	48,78%	48,85%	0,07
14/10/2020	19,23$	19,21$	0,02	48,68%	48,63%	0,05
15/10/2020	19,09$	19,09$	0,00	48,31%	48,31%	0,00
16/10/2020	19,23$	19,23$	0,00	48,68%	48,69%	0,01
19/10/2020	19,04$	19,03$	0,02	48,20%	48,15%	0,05
20/10/2020	19,02$	19,01$	0,00	48,13%	48,12%	0,01
22/10/2020	18,89$	18,91$	0,02	47,81%	47,85%	0,04
23/10/2020	18,85$	18,85$	0,00	47,69%	47,70%	0,01
26/10/2020	18,82$	18,82$	0,00	47,61%	47,62%	0,01
27/10/2020	18,97$	18,96$	0,02	48,01%	47,97%	0,04
28/10/2020	18,97$	18,95$	0,03	48,01%	47,94%	0,07
29/10/2020	19,11$	19,09$	0,02	48,36%	48,32%	0,04
30/10/2020	19,12$	19,15$	0,02	48,40%	48,46%	0,06
02/11/2020	18,98$	19,05$	0,07	48,03%	48,21%	0,18
03/11/2020	18,91$	18,88$	0,03	47,85%	47,78%	0,07
04/11/2020	18,66$	18,66$	0,00	47,21%	47,21%	0,00
05/11/2020	18,65$	18,66$	0,01	47,21%	47,22%	0,01
06/11/2020	20,17$	20,23$	0,05	51,11%	51,25%	0,14
Average	19,05$	19,06$	0,0064	48,23%	48,25%	0,0002
Stand Dev	0,313	0,323		0,805	0,834	

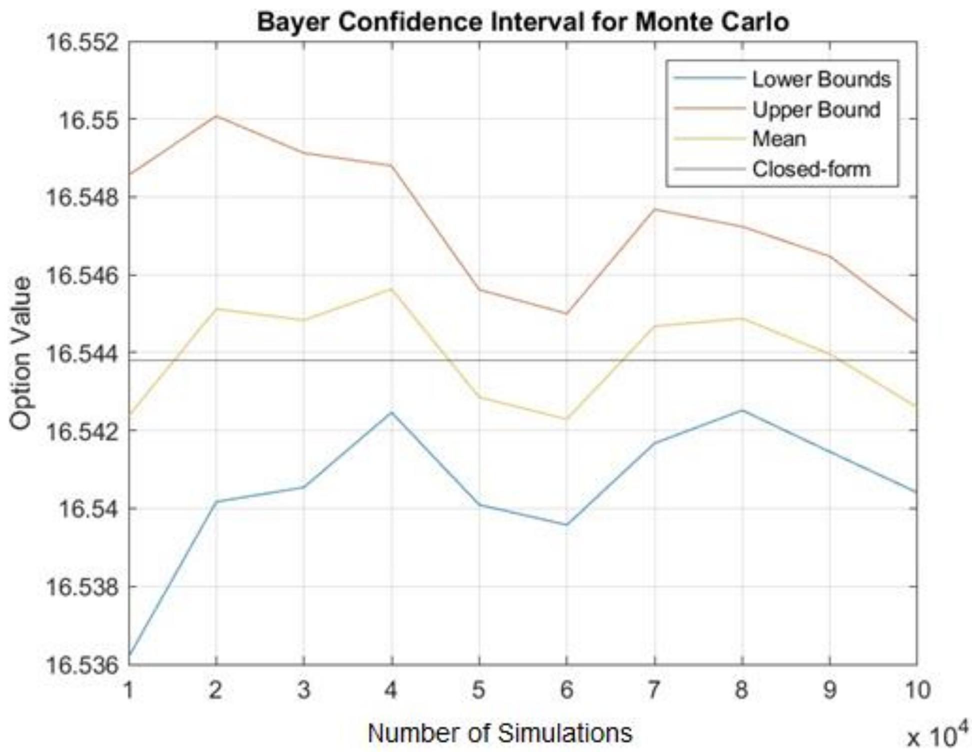
In this case, the difference between the option price is even lower, with an average deviation of 0,0064$, which represents a difference in volatility of 0,0002%. Concluding in average volatility for the period of 48,23% for the Monte Carlo and 48,25% for the Closed-Form.

Considering both strategies follow the same path and the insignificant differences from both, the usage of each of the models, becomes optional since both will get very similar results.

Monte Carlo simulation and confidence interval

The following section of the paper compares the Heston-Nandi closed-formula and a wide range of Monte Carlo simulations, for a specific date, the 26th of October of 2020, a random date from the observations above was chosen for the test.

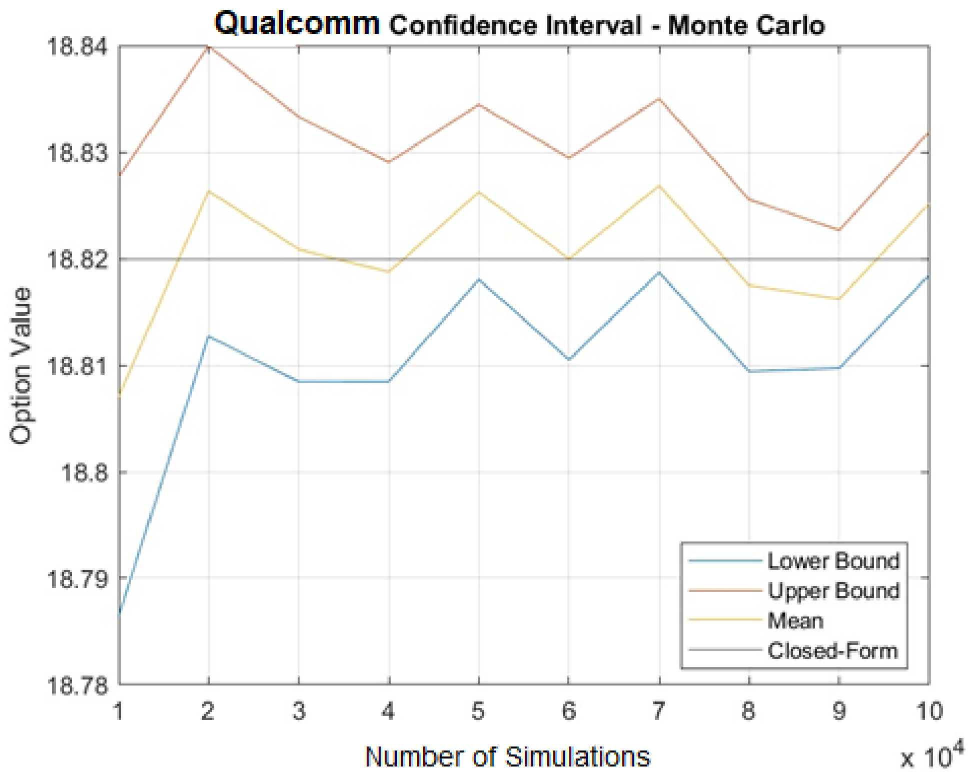
For the Bayer stock, the parameters estimated from the Heston-Nandi are the following, ω = 2,174E-21, β = 0,910, λ = 6,972E-22, α = 6,21E-05, γ = 1,985E-18. With these values, it is possible to estimate a European call option price for a maturity of 252 trading days, equivalent to one year, stock and strike price equal to 100€, for both the Closed-form and the Monte Carlo. The closed-form derivation by Heston-Nandi, gives a call option price at 18.82€, while the estimation performed by Monte Carlo depends on the number of paths simulated. Taking this into consideration, it is useful to understand how the Monte Carlo behaves as more simulations are completed as shown in the graph below.



The graph shows the 95% confidence interval for the Monte Carlo option prices for Bayer, as well as the mean value and the closed-form pricing from Heston-Nandi. As the number of simulations increases, from 10,000 to 100,000, the distance between the lower and upper bound of the Monte Carlo decreases, as expected, due to the increase in simulations, therefore, increasing the accuracy of the estimation. Furthermore, the difference between the bounds is extremely low, even with the lowest number of simulations, with only a slight variation of 0,012€. It is not essential to perform the same test for more degrees of simulations, as the deviation is significantly low and the computational requirements and time to run the Monte Carlo increases exponentially.

Additionally, the closed-form option price estimated of 18,82$, is between the 95% confidence interval, verifying the quality of the closed-form. Moreover, the mean value of the Monte Carlo follows a very similar path to the closed form, reinforcing the formulation.

Repeating the same test for Qualcomm, with the same date and specifications of option pricing estimated by the following parameters $\omega = 5{,}609E\text{-}17$, $\beta = 0{,}7599$, $\lambda = 3{,}642E\text{-}12$, $\alpha = 0{,}00022$, $\gamma = 3{,}573E\text{-}10$, the closed-form derivation gives a European call option price of 18,82$. While the graph shown below shows the confidence interval for the Monte Carlo simulations.



The 95% confidence interval for Qualcomm is 0,04$ for 10,000 simulations, a value slightly higher when compared to Bayer, however, it is explained by the volatility of the stock. The confidence interval shortens as the number of simulations increases as expected. Additionally, the closed-formula price is again between the 95% confidence interval and very close to the mean value of Monte Carlo simulations for each step.

Conclusion

The tests performed show the quality of the closed-formula by Heston-Nandi and the similarity of the results against the Monte Carlo simulation, as it follows not only the same path but also the 95% confidence interval, as the number of simulations increases from 10,000 to 100,000, showing the accurate results of the formulation by Heston-Nandi.

In conclusion, due to the lack of computational power, Banco Invest wants the lowest computationally intensive process, to save costs and time, while remaining competitive. The necessity to run a Monte Carlo for each stock, maturity, and moneyness desired, becomes an issue for the Bank, as the time consumed for all these processes are quite demanding for the computational capabilities of Banco Invest. Therefore, the problem could be solved by using the Closed-Form solution, reducing computational time without compromising the competitiveness side of the bank.

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